Programming Quantum Computers (Modules II: AA)

(**Subtrack of** Quantum Computing: An App-Oriented Approach)

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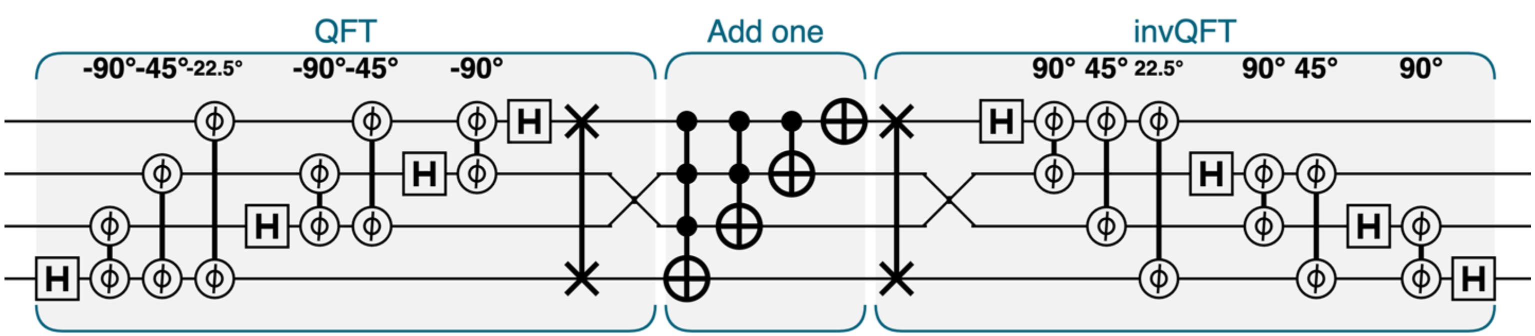
Quantum Computers are Real

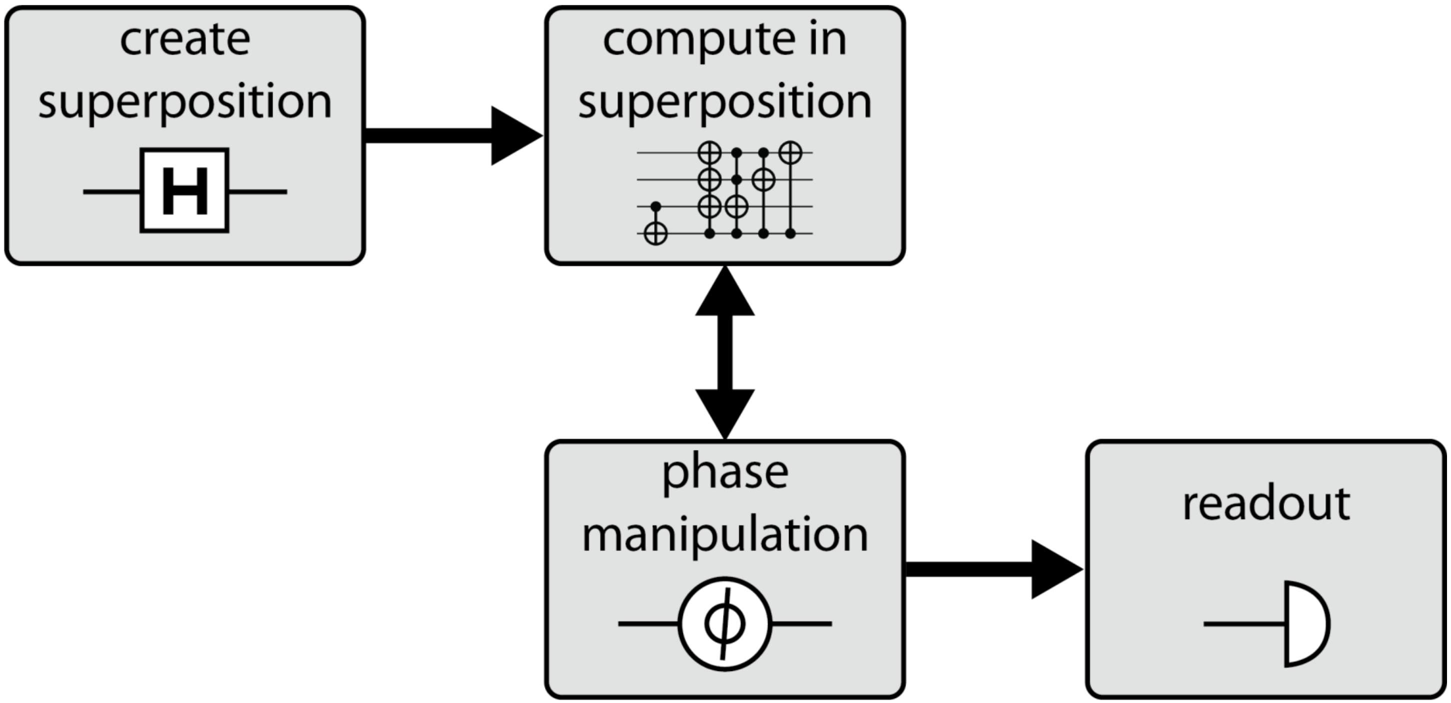
- What are they <u>useful</u> for?
  – Let's discover, by programming them!

- A hands-on approach to programming QCs/QPUs.
  – By doing; i.e., by writing code & building programs.
  – Using simulators, since real QCs are harder-to-access (so far).

- Goals: Read, understand, write, and *debug* quantum programs.
  – Ones like the following.

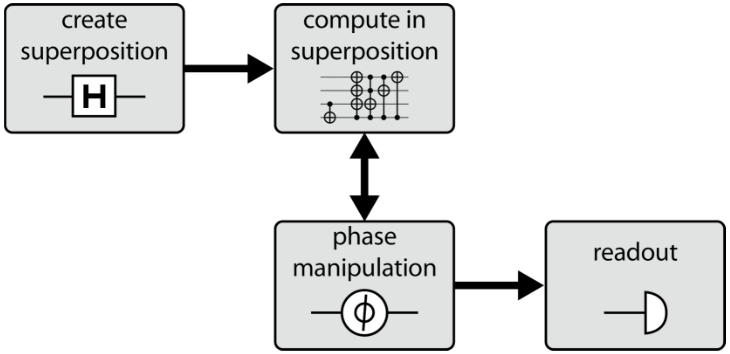


Structure of Quantum Apps



Structure of Quantum Apps

- Tendency to such structure, very roughly.
- Compute in superposition.
  - Implicit parallelism.
- Phase manipulation.
  - Practicality. Relative phase info directly inaccessible (unREADable).
- Modules are combined (*composed*) to define full quantum application.
  - Possibly in *iterations*.
- Quantum programming is an art (too).



Quantum Modules Covered

Module	Type
Digital arithmetic and logic (AL)	Compute in superposition
Amplitude amplification (AA)	Phase manipulation
Quantum Fourier transform (QFT)	Phase manipulation
Phase estimation (PE)	Phase manipulation
Quantum data types (Sim)	Superposition creation

PHASE MANIPULATION MODULES

AMPLITUDE AMPLIFICATION (AA)

Lecture Outline

- Amplitude Amplification (AA): Revealing hidden info.
  - Converting phase info into magnitude info.
  - Modules `flip` and `mirror`.

- Improving probability.
  - Periodic probabilities!
  - Ideal # of AA iterations.

- Multiple flipped-phase (“marked”) values.
  - Ideal # of AA iterations.

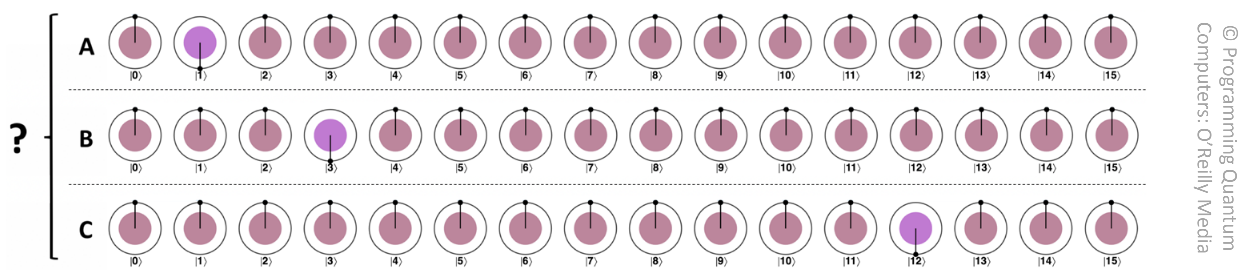
- Uses of AA.
  - QSE, SAT, and optimization problems.

- Why is `mirror` called a mirror?
  - “Slingshotting”.

Converting Phase To Magnitude

- Computing in superposition is useless if we can't actually READ computation results (Quantum parallelism).
- Amplitude amplification converts inaccessible *phase* differences into READable *magnitude* differences (and vice versa).
  - Simple, elegant, powerful, and very useful.

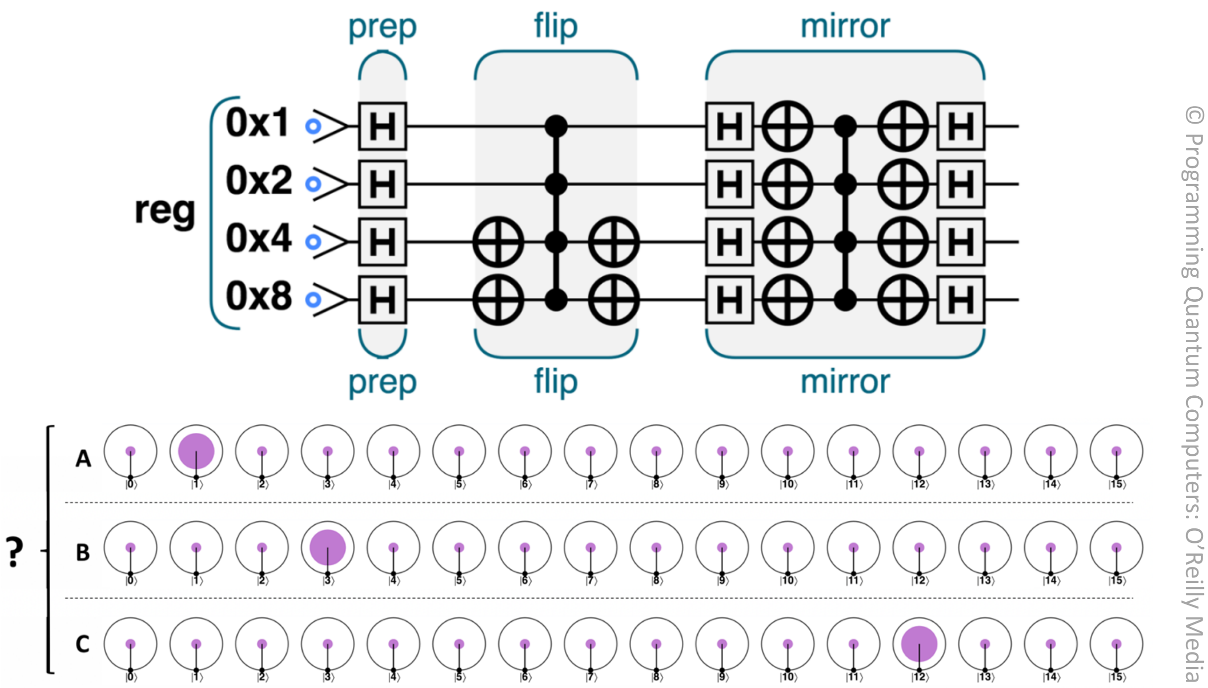
- Consider the following three states of 4 qubits:



  - States *A*, *B* and *C* only differ in the phase of one value: a *marked value*.

Converting Phase To Magnitude

- Reveal the hidden: Using `mirror` (and `flip`).

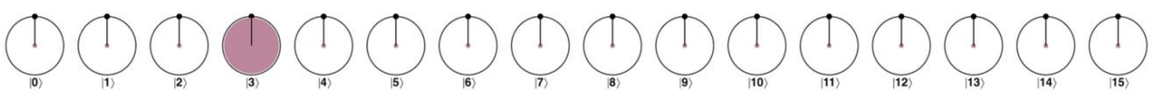


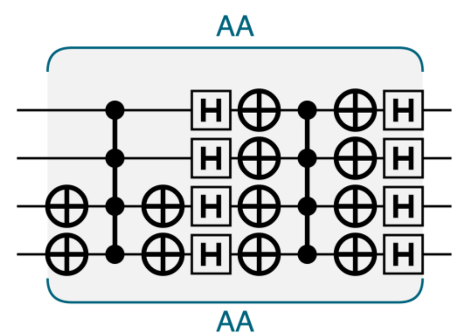
- Marked value has a READ probability of about 47.3%; nonmarked values of about 3.5%. Still not great! We can do better.

Improving Probability

- Repeating `mirror` doesn't help.
  - Doesn't improve probability, but rather returns to original state (starting point).
  - `mirror` is its own inverse (`mirror`^{2} = `no-op`).

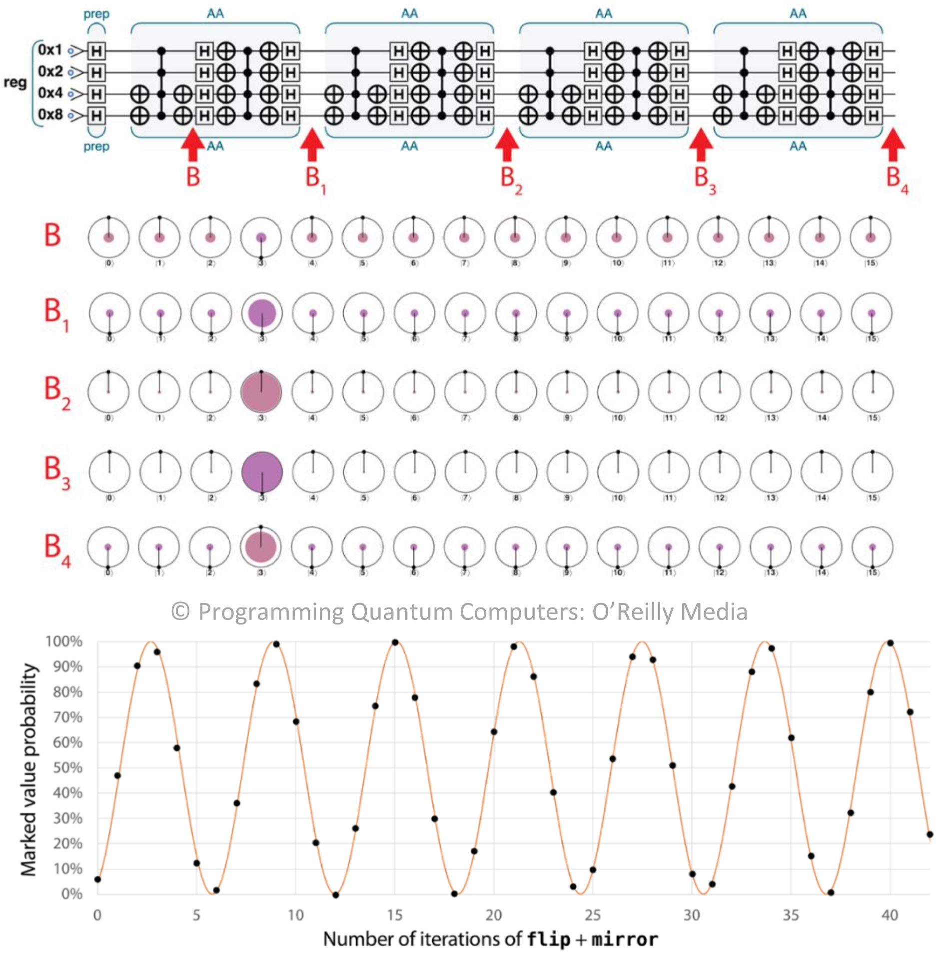
- Solution: Repeat `flip` and `mirror`.
  - AA (amplit. amplif.) iteration: 47% → 90%.





  - Three iterations: 90% → 96%.
  - Monotonically increasing? More iterations?
    - No! … Probability in fact decreases. How, and Why?

Periodic Probabilities!

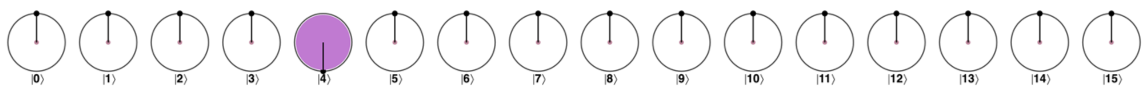


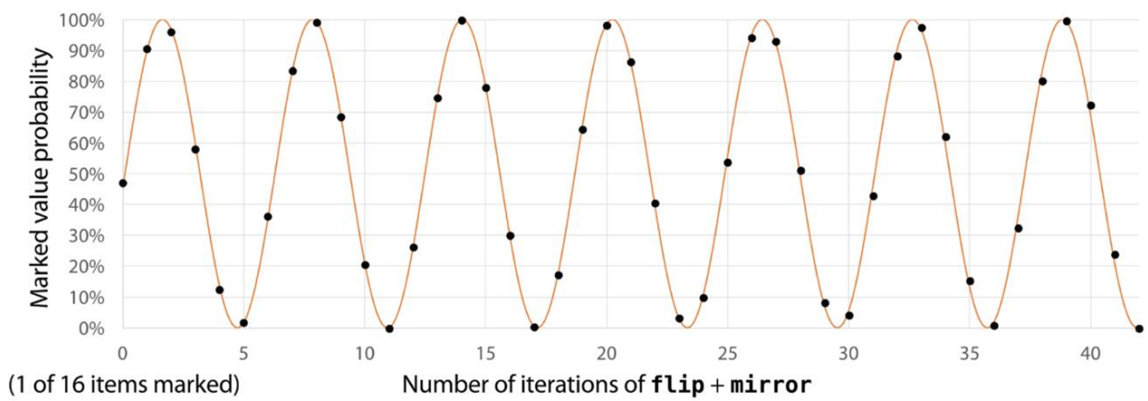
$$N_{AA} = \left\lfloor \frac{\pi}{4} \sqrt{2^n} \right\rfloor$$

(highest in <u>first</u> oscil.)

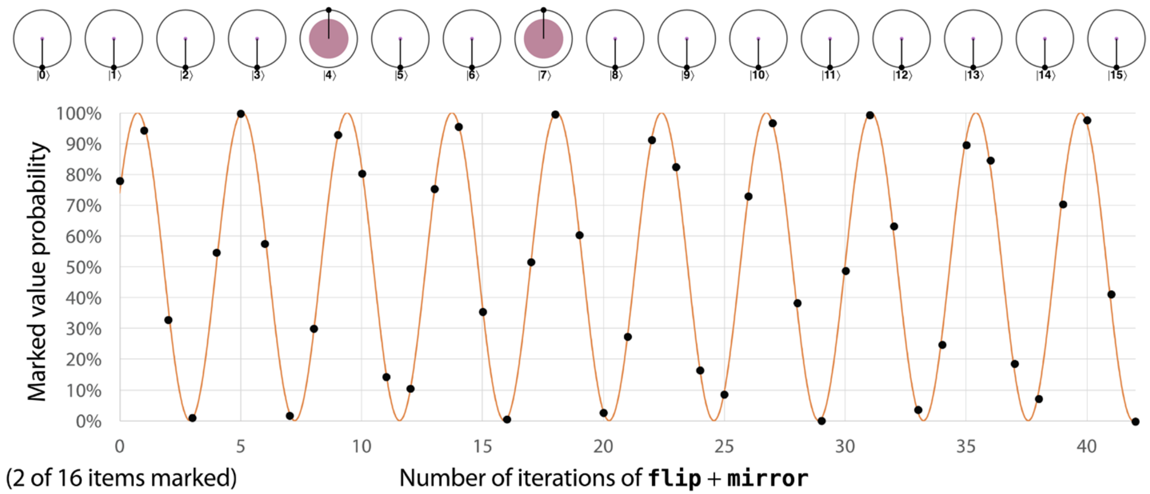
Multiple Flipped-Phase Values (1)

- Using arrays and `for` loops, we can vary the number of flipped phases.
- Starting with one phase, we reproduce earlier results.





Multiple Flipped-Phase Values (2, 3; Frequency Increases)



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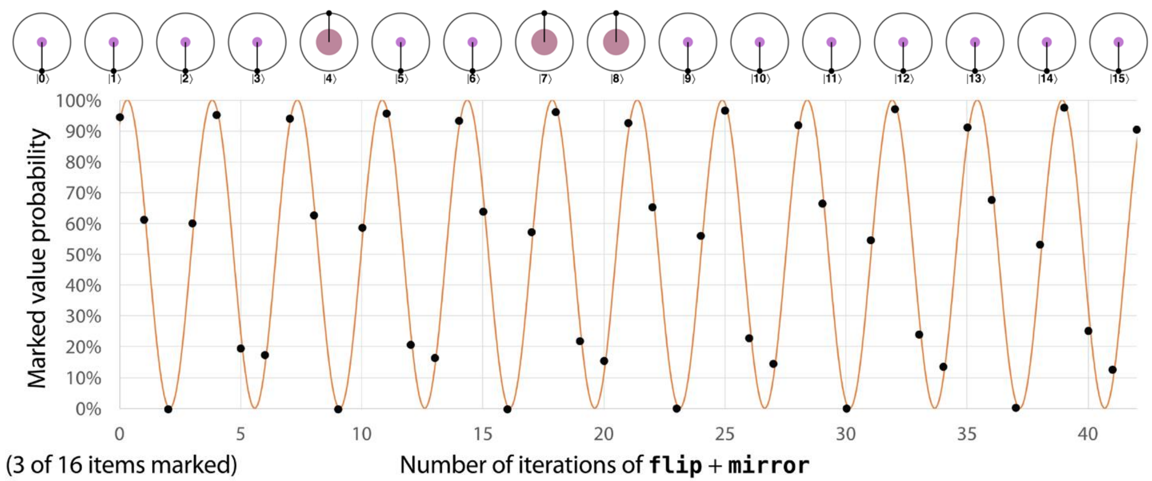
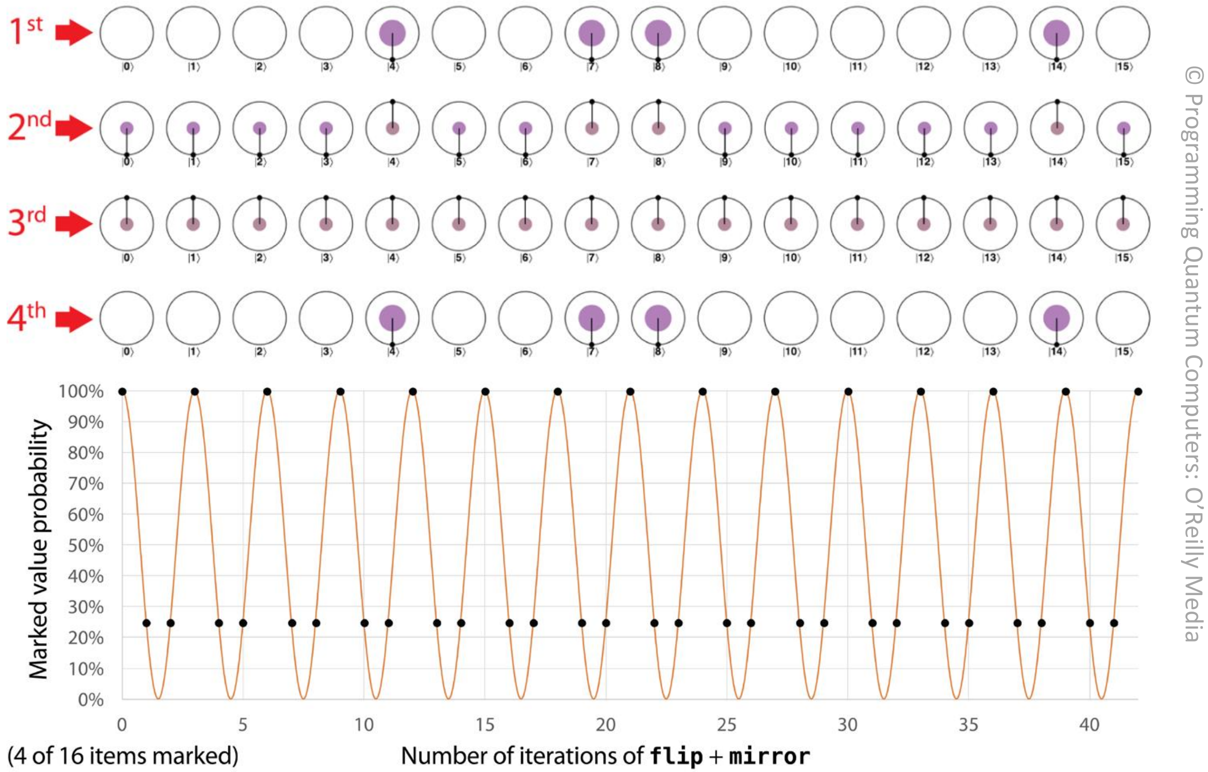


$$N_{AA} = \left\lfloor \frac{\pi}{4}\sqrt{\frac{2^n}{m}} \right\rfloor$$

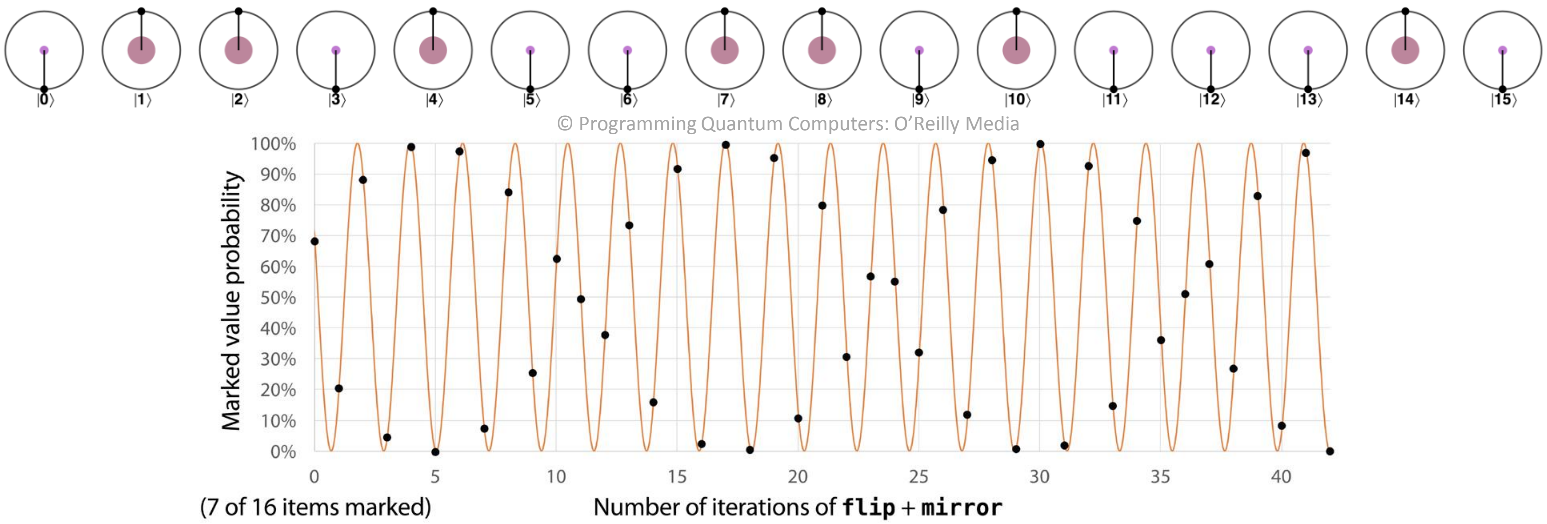
Multiple Flipped-Phase Values (4)



- Probability of READing a marked value repeats every third iteration.
  - The very first iteration gives us 100% probability of success.

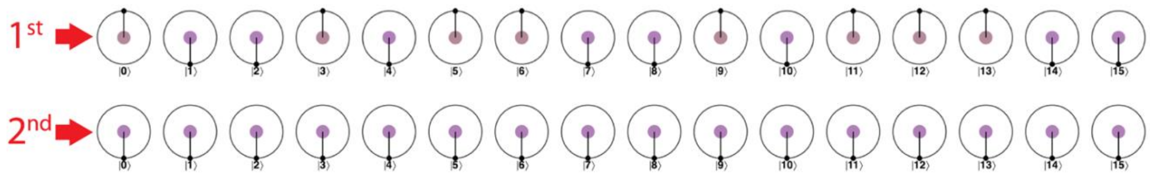
Multiple Flipped-Phase Values (7)

- Frequency continues increasing.



Multiple Flipped-Phase Values

- Coming to a halt, at 8 (i.e., 50%) marked values.
  - No need for AA iterations (useless).



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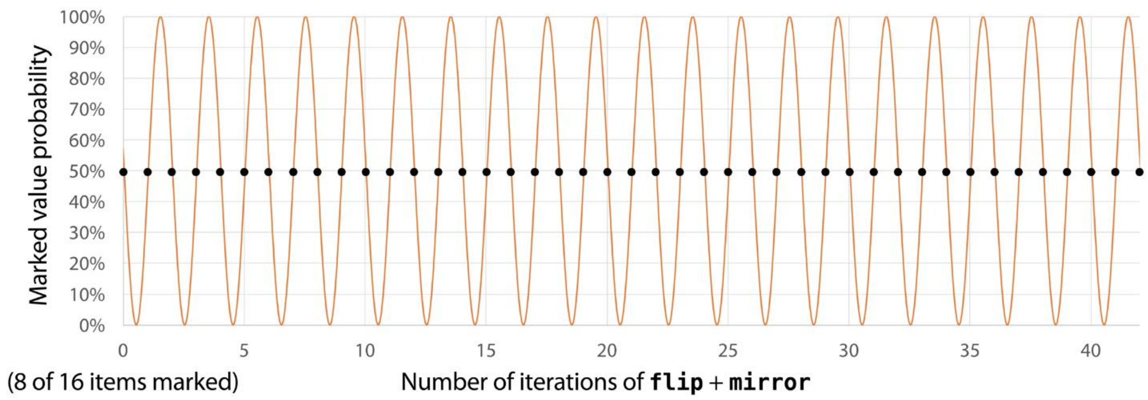


Using AA

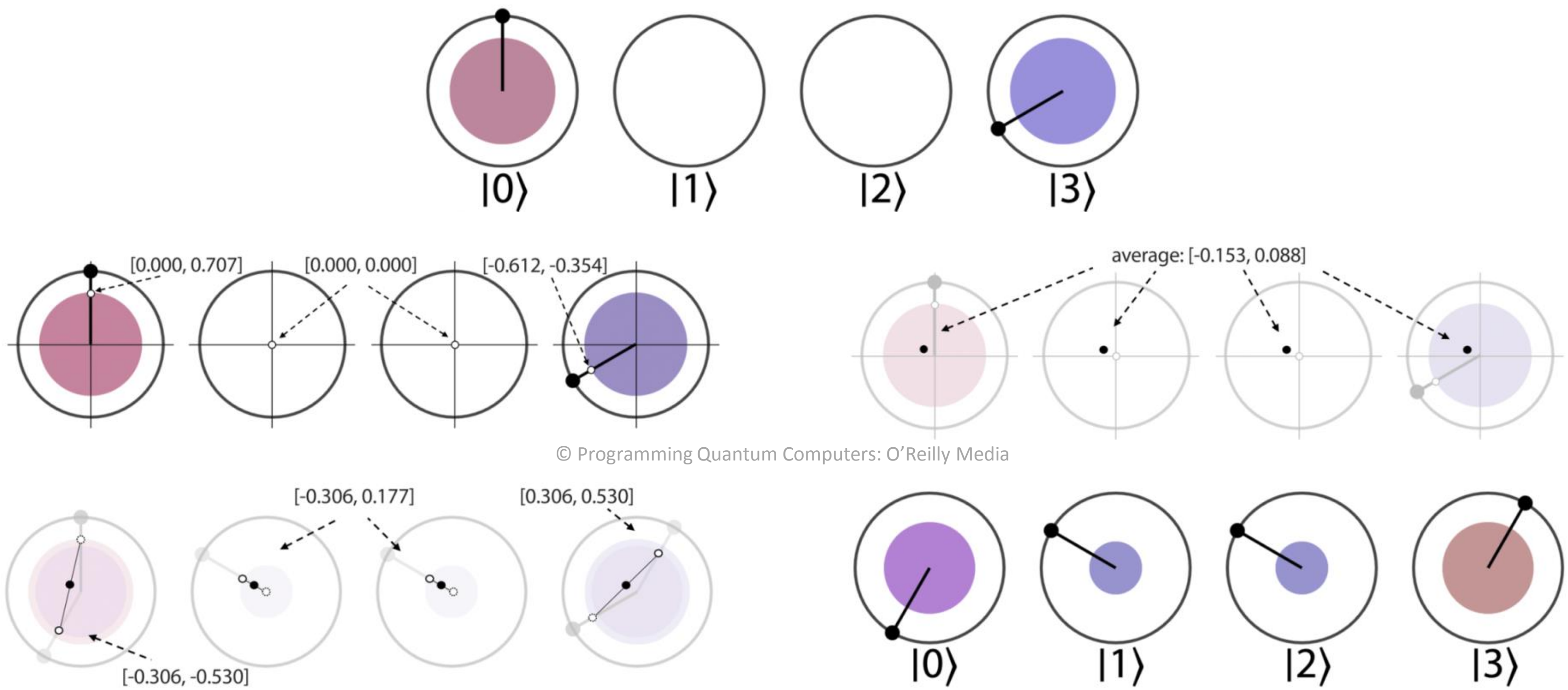
- Converting unREADable phases into READable magnitudes sounds useful…
  - But how to actually employ this?

- Quantum Sum Estimation (QSE)
  - A “feeling” for QSE. (More details later.)
  - Frequency increases with number of marked (flipped) values.
  - QFT (next week’s class) can read frequencies.
  - A form of QSE: Combining AA and QFT we can know *how many values* are marked.
  - More details on QSE in Ch. 11.

- AA can be speedup algorithms that invoke a subroutine that *repeatedly* checks the validity of a solution.
  - E.g., Boolean satisfiability (SAT), and finding global and local minima (or maxima).
  - In apps, the `flip` (value marking/checking validity) module changes, while the `mirror` module remains the same.

Why is `mirror` so-called?

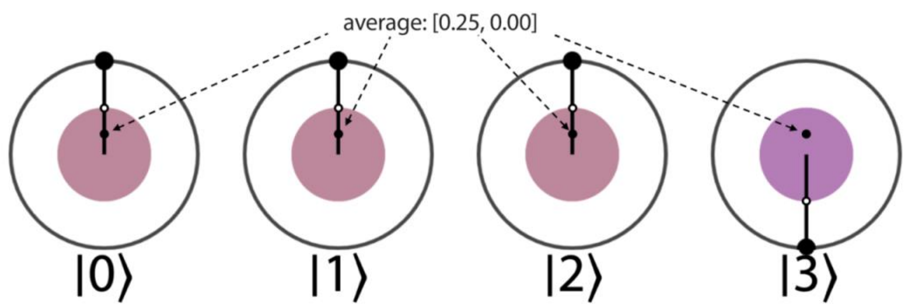
- How does `mirror` change a phase difference into a magnitude difference?

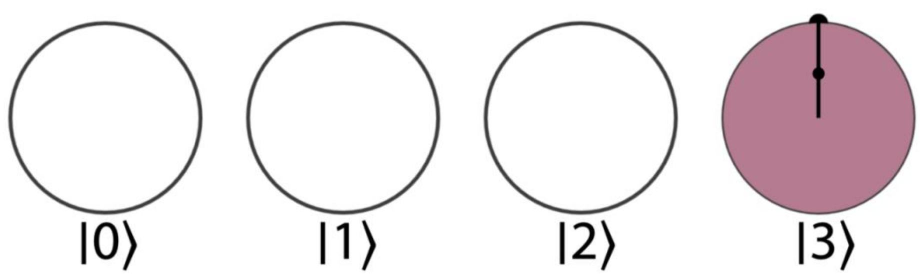


Average is the same. Reapplying `mirror` reverses its action.

“Slingshotting”

- One oddball…
  - Very common case in practice.





Discussion

Q & A

Research Question

- Now, that we know precisely how `mirror` works, can we suggest a better info-revealing module?
  - i.e., a module that reveals phase info with, e.g., lesser iterations, or lesser operations, or that produces larger magnitudes.
  - If you think it is possible, suggest *how*.
  - If you think it is not possible, explain *why* not.

Homework

- Implement multi-AA on QX, Q#, and Cirq.

Next Lectures APPetizer!

- In next lectures:
  - Ch.7: Quantum Fourier Transform (QFT).
    - Most important textbook chapter (based on all earlier ones, and is basis for all next ones).
  - Ch.8: Phase Estimation.
    - Most mathematically-challenging module.
  - Then… we move on to APP(lication)s.
    - Ch.9: Quantum data types, QRAM, and quantum simulation.
    - Ch.10: Quantum search (Grover's algorithm).
    - Ch.11: Quantum graphics (Supersampling).
    - Ch.12: Quantum cryptography (Shor's algorithm).
    - Ch.13: Quantum machine learning (QML).

  - Reading Ch.7 and Ch.8 *before* classes is a MUST.

Course Webpage

http://eng.staff.alexu.edu.eg/staff/moez/teaching/pqc-f19

- Where you can:
  - Download lecture slides (incl. exercises and homework).
  - Check links to other useful material.

Thank You